DYNAMICS

Sheet 8

1. Consider the dynamical system

$$\dot{x} = y + \varepsilon x(1 - x^2 - y^2)$$

$$\dot{y} = -x + \varepsilon y(1 - x^2 - y^2)$$

where ε is a positive constant. By changing to polar coordinates r, θ show that

$$\dot{r} = \varepsilon r(1 - r^2) \text{ and } \dot{\theta} = -1.$$

Hence find r and θ in terms of time t, and sketch the phase paths.

2. A particle moves on the smooth inside surface of the hemisphere $z = -\sqrt{a^2 - r^2}, r \leq a$, where (r, θ, z) denote cylindrical polar coordinates, with the z-axis vertically upward. Initially the particle is at $z = 0$, and it is projected with speed V in the θ-direction. Show that the particle moves between two heights in the subsequent motion, and find them.

   Show, too, that if the parameter $\beta = V^2/4ga$ is very large then the difference between the two heights is approxiamately $a/2\beta$.

3. A particle of unit mass moves in a plane under a central force $\dfrac{\lambda^2}{r^5}$, where λ is a constant. Explain why

$$\dot{r}^2 + r^2\dot{\theta}^2 - \frac{\lambda^2}{2r^4} = \text{constant}.$$

   The particle is projected from a point P at which $r = a$ with speed $\lambda/\sqrt{2}a^2$ along a line through P. Find a differential equation for r as a function of the polar angle θ along the orbit and show that the orbit is a circle through O.

   [Note that this is true regardless of the initial *direction* of the motion(!)]