DYNAMICS

Sheet 7

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1. If a particle has cylindrical polar coordinates (r, θ, z), and $\theta = \omega t$, where ω is a constant, show that

$$\ddot{\mathbf{r}} = (\ddot{r} - r\omega^2)\mathbf{e}_r + 2\omega\dot{r}\mathbf{e}_\theta + \ddot{z}\mathbf{e}_z.$$

A bead of mass m slides on a smooth straight wire which is horizontal and rotates about a vertical axis through one end O with constant angular speed ω. The bead is attached to O by a spring of natural length a which lies along the wire and gives rise to a force equal to α times its extension. Show that

$$\ddot{r} + \left(\frac{\alpha}{m} - \omega^2\right) r = \frac{\alpha}{m} a,$$

where r denotes the distance of the bead from O.

Show also that an equilibrium point $r_0 > 0$ exists if $\omega^2 < \alpha/m$, and determine whether this is stable or unstable.

2. A smooth straight wire rotates with constant angular speed ω about the vertical axis through a fixed point O on the wire, and the angle between the wire and the upward vertical is constant and equal to α, where $0 < \alpha < \frac{1}{2}\pi$. A bead is free to slide on the wire. Show that $z(t)$, the height of the bead above O, satisfies the equation

$$\ddot{z} - \left(\omega^2 \sin^2 \alpha\right) z = -g \cos^2 \alpha$$

and find $z(t)$ if $z(0) = h$ and $\dot{z}(0) = 0$. Deduce that if $h > g \cot^2 \alpha/\omega^2$ then $z(t) \to \infty$ as $t \to \infty$ but if $h < g \cot^2 \alpha/\omega^2$ then $z(t) \to -\infty$ as $t \to \infty$.

3. Find the equilibrium points of the system

$$\begin{aligned} \dot{x} &= -x + xy, \\ \dot{y} &= -y + xy, \end{aligned}$$

and determine whether they are stable or unstable, according to linearized theory.

Sketch the phase paths in the neighbourhood of the equilibrium points.